Examining the TPACK and the Utilization of Mathematical Action Technology with Dynamic Geometry Software: A Case of Prospective Mathematics Teacher

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ABSTRACT

Mathematics teachers are encouraged to use technology like dynamic geometry software tools in their lesson with regard their curriculum as they become more widely available in schools. This qualitative study investigates prospective mathematics teachers who produced lesson video with dynamic geometry software using Technological Pedagogical Content Knowledge (TPACK) and mathematical action technologies. The findings demonstrated that prospective mathematics teachers successfully used dynamic geometry software applications and pedagogical practices as reinforcing tools. To support and expand the findings, participants taught mathematics 50% of the time using dynamic geometry software as conveyance technology. In addition, they employed mathematical technology to balance the elements of TPACK in their solutions almost 50% of the time. For some, mathematical tools can help grow virtual students' mathematical knowledge. This study recommends that higher educational programs needs to empower educator in creating a learning cycle for pre-service teachers to design, implement, and reflect on creating video lessons using dynamic geometry software or other mathematical action technology as well as to improve education quality.

Keywords: TPACK, Mathematical Action Technology, Dynamic Geometry, Educational Technology, Education Quality

INTRODUCTION

Technology plays an essential role in education. The rapid development of technology requires the education world to adapt to these changes. The government has launched technology-integrated learning (Faiz et al., 2022). It is in response to the needs of the industrial revolution 4.0, where humans and technology are aligned to create new opportunities creatively and innovatively (Rahayu, 2021). Therefore, the purpose of Education 4.0 is to prepare human resources (HR) that are creative and follow current demands when the world is facing a digital-based industrial revolution (Komang et al., 2022).

Based on Law Number 14 of 2005 concerning Teachers and Lecturers, teachers must possess four competencies. The four competencies are: pedagogic, personality, social, and professional. The era of globalization with rapid technological development requires teachers not only to master pedagogical competence and material content but also competence in technology. According to Nasution (Nasution, 2018), technology can provide benefits in learning such as: 1) improving students' focus, concentration, motivation, and independence, and 2) helping teachers reduce time in delivering materials, creating a more interesting learning experience for students, facilitating the design of more interesting materials, and encouraging teachers to improve their understanding and computer skills. In addition, according to Akhwani

& Rahayu (2021), some of the objectives of using technology in learning are to improve learning quality, student satisfaction, and reach learning goals.

TPACK (Technological Pedagogical Content Knowledge) is a conceptual framework that integrates three types of knowledge, namely: technological, pedagogical, and content knowledge; to support the use of technology in learning (Mishra & Koehler, 2006). The development of TPACK is critical for teachers to conduct effective technology-integrated learning (Mishra & Koehler, 2008). TPACK is particularly important for prospective mathematics teachers because technology has become an integral part of everyday life and can influence the learning process in the classroom.

One of the technologies that can be utilized in learning mathematics is Mathematical Action Technology (MAT). MAT is software for creating mathematical animations, simulations, and demonstrations to help students understand mathematical concepts better (Bonafini & Lee, 2021). MAT can allow students to visualize abstract and complex mathematical concepts. However, the use of MAT in mathematics learning is still not widely used in Indonesia, and not all prospective mathematics teachers have sufficient knowledge and skills in using MAT (Nur, 2017). One of the causes is the need for teacher attention to technology content knowledge which is one of the TPACK components in teachers (Mishra & Koehler, 2008). Therefore, it is necessary to explore the TPACK of prospective mathematics teachers in making learning videos using MAT.

GeoGebra Classroom, a dynamic geometry software, has emerged as a promising tool for teachers and students in secondary school mathematics. Its ability to be used on the same device in different classes makes it even more appealing and complementary to teachers' pedagogical approaches (Sutopo & Ratu, 2022). By providing teachers with the opportunity to use GeoGebra Classroom, they can experience conveyance and mathematical action technologies in an integrated way (Zöchbauer et al., 2021). This study aims to identify the level of TPACK of prospective mathematics teachers in making learning videos using MAT. The results of this study can contribute to developing a more interactive and creative mathematics learning curriculum using technology. In addition, the results of this study can also provide input for universities that offer mathematics education study programs to improve the quality of education for prospective mathematics teacher students.

METHODS

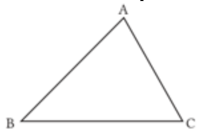
A qualitative approach was taken for this study, and thematic analysis was employed (Vaismoradi et al., 2013). The main goal was to comprehend a specific phenomenon or delve into intricate matters comprehensively (Vaismoradi et al., 2013). The study involved gathering videos and other materials submitted by participants. The researchers then identified instances of technological, pedagogical, and content knowledge demonstrated by the participants and their use of mathematical action tools.

Participants

The study involved students who were mathematics teacher candidates in their sixth semester and had undergone various pedagogical courses, such as assessment, problem-solving, realistic mathematics education, and micro-learning. A total of 25 students were part of the study, and the researchers utilized purposeful sampling to choose cases that were information-rich (Cresswell & Plano Clark, 2011). For this paper, the focus is on three chosen participants: HYA, MIF, and RTO. All the students had completed their school mathematics education program and were expected to utilize their mathematical and pedagogical knowledge acquired through previous content and methods courses.

The learning video project was introduced as one of the main projects in the microlearning course's final project to allow the participants to implement their pedagogical, mathematical, and technological knowledge in an integrated manner to develop their TPACK. The project is divided into two sub-sections: (1) selecting a math problem as shown in Table 1 and planning a video lesson on how to solve the problem for junior high school students using mathematical action technology, and (2) creating a 20-minute video lesson using Zoom Meeting or Google Meet.

Table 1. The Problems Selected by the Participants

Participants	The selected problem
HYA	A rectangle is constructed by connecting four points, with the coordinates of each being A(-1,0), B(3,3), C(6,-1), and D(2,-4). Show that the rectangle $ABCD$ is a square.
MIF	Find the sum of the angles in a regular polygon with n angles, and show that it is (n - 2) × 180°.
RTO	List all the possible ways to draw $\Delta A'B'C'$ that are congruent to ΔABC? 

Data Sources

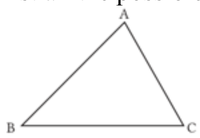
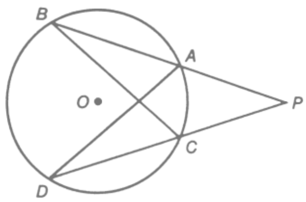
The data sources for the study consisted of two parts: (a) lesson plans created by the participants that contained their solutions to the problem, and (b) video lesson files created by the participants. The researchers primarily used the video lesson files as the primary data source to answer the research questions, while the lesson plans were used as the secondary data source.

Data Analysis

The researchers studied participants' video lessons and lesson plans to identify TPACK components and instances where the video lessons showed varying levels of mathematical action tool implementation, such as amplifiers and reorganizers. Table 2 shows the codes for TPACK components (DeCuir-Gunby et al., 2011), their definitions, examples, and data sources used by the researchers. Meanwhile, Table 3 shows the code for the mathematical action technology (MAT) level that the participants carried out. For a complete view of Table 2, refer to the supplementary material.

Table 2. Coding Framework for TPACK in Lesson Plans

Indicators	Component
CK "Knowledge about actual subject matter that is to be learned or	**Problem 1** Find the sum of the angles in a regular polygon with n angles and show that it is (n - 2) × 180°. **Problem 2**

Indicators	Component
taught." (Mishra & Koehler, 2006, p. 1026). Indicators of CK include actions such as: - solving mathematical problems correctly **(CK1)** - Proving mathematical statements deductively **(CK2)** - Communicating mathematical ideas effectively **(CK3)** - Using mathematical representations properly **(CK4)**	A rectangle is constructed by connecting four points, with the coordinates of each being A(-1,0), B(3,3), C(6,-1), and D(2,-4). Show that the rectangle $ABCD$ is a square. **Problem 3** List all the possible ways to draw $\Delta A'B'C'$ that are congruent to ΔABC?  **Problem 4** A parallelogram $ABCD$, its two diagonals intersect at point E. Describe the isometry that can map triangle ΔAED to triangle ΔCEB. Prove that triangle ΔCEB is a transformation of triangle ΔAED. **Problem 5**  Show that $BP.\ AP = DP.CP$ Note: Participants are asked to choose at least 1 problem they want to solve with Dynamic Geometry Software to create a lesson plan.
PCK PCK means representing the "pedagogy that is applicable to the teaching of specific content" (Koehler & Mishra, 2005, pp. 133–134). Indicators of PCK include actions such as: - Explanations about solving the problem using mathematical pre-requisite knowledge **(PCK1)** - Attention to address of common misconceptions **(PCK2)** - Explanations of different approaches to solving a problem **(PCK3)** - Use of multiple representations **(PCK4)**	a) Identify the misconceptions that may arise when students work on these problems! b) How can you prevent these misconceptions from arising?
TPACK TPACK means "knowledge required by teachers for integrating technology into their teaching in	a) What mathematics learning technologies do you use to teach these problems to students? (Give as many examples of technologies you can use as possible) b) Why did you choose this technology over others? Explain in more detail! c) What are the steps you use to teach students to solve the problem you have chosen? (Create a lesson plan) d) Make a video of how you teach the problem with the situation and technology you have chosen.

Indicators	Component
any content area" (Schmidt et al., 2009, p. 125) Indicators of TPACK include actions such as: - Combining mathematics, technology, and teaching **(TPACK1)** - Using strategies that integrate content, technologies, and teaching approaches.	

Table 3. Coding framework for mathematical action technology (MAT) level

Level	Description	
0	Conveyance technology	Amplifier
1	Mathematical Action tool used as screenshots	Amplifier/Organizer
2	Mathematical Action tool used with manual enhancement	
3	Mathematical Action tool used with manual enhancement	
4	Mathematical Action tool used in its native form or stop-motion	

Researchers analyzed video excerpts of participants to extract insights on the TPACK components. The same method was employed to determine the different levels of implementation of mathematical action tools. Using the TPACK framework allowed researchers to cross-check each coded data excerpt from the video files with the participants' lesson plans. The researchers compared the participants' reflections on their choice and utilization of mathematical tools with their levels of implementation in the videos.

The researchers coded the documents as a group, following individual coding of the same documents. During the group coding, the researchers had to agree on the codes used for each data excerpt. In the event of inconsistencies, the code with most of the evidence presented in the data excerpt was applied.

The researchers used the triangulation process described by Golafshani (2003) and Moon (2019) to validate the data during the analysis. In the final analysis stage, the researchers collaboratively created research memos for each student. These memos served as a tool for the researchers to extract meaning from the data, as Lapan et al. (2012) suggested. Finally, the research memos were compiled to form a statement on how the participants demonstrated TPACK components and utilized mathematical action technologies in their videos.

RESULT AND DISCUSSION

The results of the analyses carried out to explore how pre-service teachers demonstrated their grasp of technological, pedagogical, and mathematical knowledge while employing mathematical action technology within a video lesson are detailed in this section. The participants effectively employed their preferred dynamic geometric software in the video they produced, with a majority opting for GeoGebra. The lesson orchestrated by the participants was analyzed based on their video-lesson plan and the resulting learning video. The participants adeptly synchronized their verbal delivery, pointer movement, and visual-dynamic elements. However, it is notable that none of the participants instructed the audience to pause the video for contemplation of problem solutions. Furthermore, the participants allocated only a limited timeframe for viewers to reflect upon each posed question or problem within the video lesson.

Among the three participants (HYA, MIF, RTO), merely two participants (HYA, MIF) satisfy the prerequisites stipulated for the desired video lesson. Regrettably, RTO creates a presentation video rather than an instructional video lesson. Consequently, the video produced by RTO is excluded from the analysis. Our focus solely pertains to the exploration of TPACK, and the utilization of mathematical action technology as outlined in the lesson plan created by RTO.

Technological, pedagogical, and mathematical knowledge is presented in a learning video created by GeoGebra-assisted prospective teachers.

As illustrated in Figure 1, the participants used the screens on GeoGebra and Powerpoint in online learning to distribute knowledge visibly and orderly. The participants synchronized their voices with the information displayed on the screens. HYA and MIF kept a consistent speed throughout the video, exhibiting acceptable pedagogical understanding.

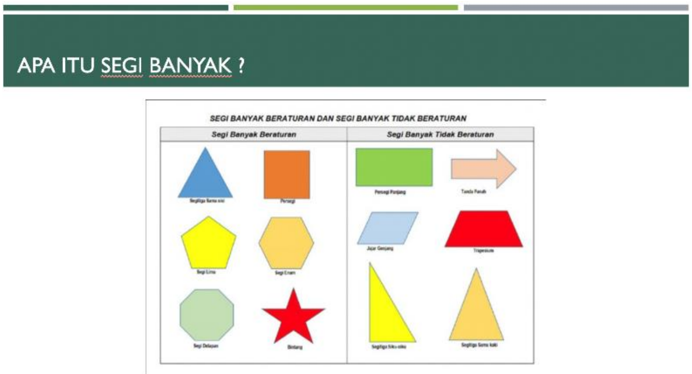


Figure 1. MIF Screen

Translate: What is a polygon?

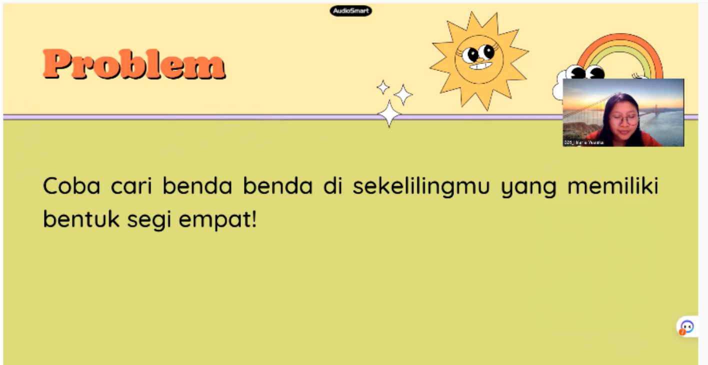


Figure 2. HYA Screen

Translate: Try to find objects around you that have a rectangular shape!

Among the three participants, MIF elaborated the information in his video sufficiently such that students could follow some of the stages and graphical representations he used in the problem apperception, as seen in Figure 1. When discussing the problem, HYA employed the "game guide mode" (See Figure 2). She played a game with the students before tackling the designated fundamental problem.

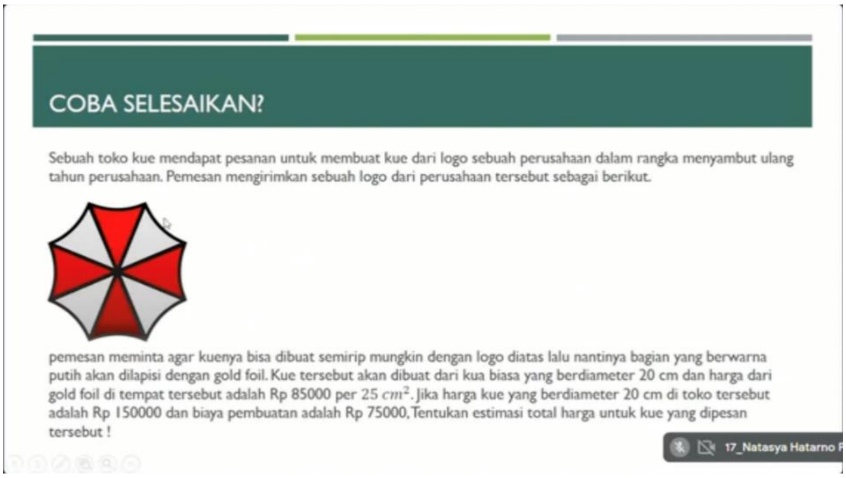


Figure 3. MIF problems in the context of logo making

Translate: A cake shop received an order to make a cake from a company logo for the company's anniversary. The customer sent a logo of the company as follows. The orderer requested that the cake be made as similar as possible to the logo above, and then later, the white part will be coated with gold foil. The cake will be made from a regular cake with a diameter of 20 cm, and the price of gold foil in that place is Rp 85000 per 25 cm. If the price of a 20 cm diameter cake at the store is Rp 150000 and the manufacturing cost is Rp 75000, determine the estimated price for the ordered cake!

Additionally, MIF illustrated the issue by relating it to real-life scenarios familiar to students (See Figure 3). MIF adapted the original math problem by incorporating the concept of logo creation. She also employed GeoGebra as a tool to address the contextually enhanced problem (See Figure 4). MIF incorporates real-world contexts to ensure that mathematical challenges are perceived not just as theoretical exercises but also as practical solutions to everyday problems.

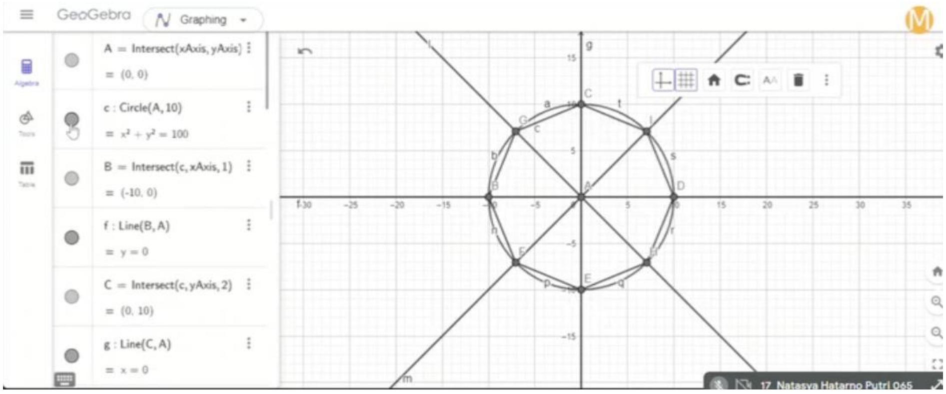


Figure 4. Problem-solving done by MIF integrated with the context of logo creation using GeoGebra.

MIF uses a contextual approach in teaching mathematics, bridging abstract concepts with real-life applications that students recognize. By linking the math problem to creating a logo, he demonstrated the relevance of mathematics in contemporary aspects such as graphic design. The reference to an image signaled a visual representation that reinforced this understanding (Porras-Hernandez & Salinas-Amescua, 2013; Tabak, 2010). Furthermore, the use of GeoGebra highlights the integration of technology in problem-solving, introducing students to tools that are beneficial for current tasks and future academic and professional opportunities (Birgin & Uzun Yazıcı, 2021; Misrom et al., 2020; Mosese & Ogbonnaya, 2021). Through this approach, MIF seeks to change students' perception of mathematics from a mere collection of formulas to a valuable skill for dealing with the challenges of everyday life.

Pedagogical-content knowledge presented by participants

Each of the participants displayed appropriate pedagogical-content knowledge within their lesson plans, albeit not all indicators of pedagogical-content knowledge were evident in the video lesson produced by HYA and MIF. Table 3 illustrates the sample excerpt that exemplifies pedagogical-content knowledge drawn from the participants' lesson plans.

Table 3. The Sample Excerpt Exemplifies PCK from the Participants' Lesson Plan

Participants	Indicators	Excerpt
HYA	**PCK1**: The participant explains the problem solution using mathematical pre-requisite knowledge.	"1. Teacher encourages students to recall information regarding rectangular prisms (here participant mistakenly used the term "solids" for quadrilateral which then means rectangular prisms). 2. Teacher requests the students to try listing any elements that are part of rectangular prisms (quadrilaterals). 3. Teacher asks students to identify the shapes of objects around them and describe whether those objects constitute rectangular prisms (quadrilaterals)." HYA employed mathematical prerequisite knowledge about quadrilaterals to address the problem of proving a square. It is worth noting, however, that a minor error is present in referring to "quadrilateral" as "solid". In the lesson plan, HYA engaged the audience by prompting them to classify polygons falling under the quadrilateral category. Additionally, HYA encouraged viewers to discern and delineate objects in their surroundings that could be categorized as quadrilaterals.
	PCK2: The participant gives attention to address of	"The potential misconceptions that could arise include: (1) Students perceive that a quadrilateral is the same as a rectangle; (2) Students perceive that the constructed quadrilateral is not a square because they understand that squares have straight edges. (Here, HYA means that a square is consistently presented with vertical and horizontal sides, while its diagonal

	common misconceptions.	is depicted obliquely. Squares are seldom portrayed with slanted sides and diagonals that are depicted horizontally and vertically respectively.); (3) Students perceive that the quadrilateral is a rectangle when they only estimate the coordinates of each point; (4) Students perceive that the constructed quadrilateral is a rhombus due to its resemblance to the shape of a rhombus; (5) Students perceive that the angles formed by the quadrilateral are not right angles because the shape does not resemble the right angles commonly understood by students; (6) Students perceive that the quadrilateral is not a square because they struggle to determine the lengths of its sides; (7) Students are confused in determining which coordinates are for x and y, which can lead to the inversion of point placement." "The solutions are: (1) Providing a clear definition of shapes that are quadrilaterals. Additionally, explaining the characteristics of each shape of every quadrilateral category, highlighting their differences and similarities; (2) Offering concrete examples of quadrilaterals. Granting students the opportunity to identify and locate square, rectangle, and rhombus shapes in their surroundings to deepen their understanding; (3) Teaching students how to measure the length and width of flat shapes using measurement tools like rulers or calipers; (4) Stimulating critical and creative thinking abilities in students, demonstrating that a square remains a square whether it is rotated 45 or 90 degrees; (5) Utilizing diverse teaching methods such as visuals, videos, or educational games to engage students in a more captivating and enjoyable learning process regarding square and rectangle concepts; (6) Encouraging students to inquire and engage in discussions about square and rectangle concepts, enabling the accurate identification and correction of emerging misconceptions; (7) Providing ample exercises to ensure that students can distinctly differentiate between squares and rectangles." HYA predicts the possible misconceptions that may emerge in the students. She writes the possibility and the solutions to address the misconceptions.
	PCK3: The participant explains different approaches to solving a problem, and use of multiple representations.	"(5) Utilizing diverse teaching methods such as visuals, videos, or educational games to engage students in a more captivating and enjoyable learning process regarding square and rectangle concepts." "2. Implementing a range of activity options: (a) Visualization and Geometric Manipulation: Utilize geometric manipulatives such as cubes, square paper, or tangrams to visually represent and manipulate square shapes. The teacher encourages students to construct squares using these manipulatives, fostering exploration of various properties and characteristics of squares; (b) Map-based Games: Task students with connecting points on a map within a certain region, leading them to identify the resulting flat shapes formed by connecting these points. This activity combines geographical understanding with geometric concepts; (c) Exploring Square Properties: Guide students in investigating square properties through activities like measuring side lengths, identifying right angles, and discerning characteristics of squares, rectangles, and rhombuses. By comprehending these properties, students can develop a deeper understanding of squares; (d) Square Construction Project: Assign students a project where they design and create models or structures using square shapes. For instance, they could craft a mini house using square blocks or design a garden with square plots. Such projects engage student creativity while reinforcing their understanding of squares; (e) Square Puzzles: Craft puzzles or riddles centered around squares as the main element. Students must solve these puzzles or assemble the pieces to form a square. This approach fosters problem-solving skills and deepens their grasp of square shapes." In the lesson plan she made, HYA suggests the integration of various approaches as students work through the problem. She advocates the utilization of multiple representations, including visuals, videos, and educational games. She cites examples such as employing geometric manipulation visualizations, utilizing a map-based game, engaging in a

		game aimed at identifying square properties, assigning a project for students to construct a square, and employing a square puzzle activity.
MIF	**PCK1**: The participant explains the problem solution using mathematical pre-requisite knowledge.	"Opening Phase: (1) Commence with a greeting; (2) Recapitulate the concept of regular polygons; (3) Inquire about the total of interior angles' measurements in a triangle and a quadrilateral; (4) Pose a prompting question such as: 'How about the total of interior angles' measurements in a pentagon? And (what about in) a decagon?'" In points 2 to 4, MIF assists students to retrieve their prior knowledge, which serves as a basis for resolving the problem. The prerequisite knowledge according to MIF encompasses the understanding of regular polygons and their associated interior angles' measurements.
	PCK2: The participant gives attention to address of common misconceptions.	"a) Identifying Misconceptions: One potential misconception that may arise is related to the angles within each regular polygon. Students might initially adopt an inductive approach based on their prior knowledge, assuming that the interior angle measures in a regular triangle are 60° and in a square are 90°. Consequently, they may believe that for subsequent polygons, each angle would increase by 30° compared to the previous shape." MIF anticipates a potential misconception that could emerge. Students might apply inductive reasoning that could lead them to a misconceived notion.
	PCK3: The participant explains different approaches to solving a problem, and use of multiple representations.	"It is essential to reiterate the definition of a regular polygon as a shape with equal-length sides and congruent angles. In the case of triangles, an equilateral triangle fits this criterion; for quadrilaterals, it's a square, and so on. Using images, we should illustrate which shapes qualify as regular polygons and which do not. Following this, students should be taught the process of constructing regular polygons. For instance, consider the method to construct a regular pentagon. Begin by drawing a circle, then create five lines dividing the circle's area into equal segments (using a protractor or other aids). Connect each pair of adjacent intersection points (forming chord segments) and the circle. The shape formed by these points will be a regular pentagon." MIF suggests employing a manipulative visual approach as a solution to address the misconception. Students are asked to manipulate their drawings to comprehend the properties of regular polygons.
RTO	**PCK1**: The participant explains the problem solution using mathematical pre-requisite knowledge.	"Activity 1: Recalling the topic of transformations, including reflection, translation, and rotation. The teacher provides a concise explanation of the concepts, formulas, and examples for reflection, translation, and rotation." RTO suggested that students retrieve their previous understanding of transformations, which could be applied to solve the problem at hand. RTO recommends utilizing transformations to address problems related to congruency. However, the learning activity appears to shift its focus towards transformations rather than congruency.
	PCK2: The participant gives attention to address of common misconceptions.	"Misconceptions often arise when students misunderstand the criteria for two geometric shapes being considered congruent. This is partly due to the resemblance of congruency conditions to similarity. This similarity can lead to confusion and occasional mix-ups among students. Below are potential misconceptions that may arise during the problem-solving process: (1) Students assume that two congruent plane figures are necessarily similar, and two similar plane figures are not necessarily congruent; (2) Students believe that congruent triangles are not necessarily similar, and two similar triangles are automatically congruent; (3) Students think that if two plane figures have corresponding equal angles and sides, they are neither congruent nor similar; (4) Students get inverted while matching corresponding sides when the image is flipped or rotated. To address those potential misconceptions that may arise, initially, I provide an initial example by showcasing real-world objects present around the students. The objects I present represent the concept of congruence. Once the students have a grasp, I proceed to explain the definition and conditions of congruence, followed by introducing additional examples. To distinguish from similarity, I also give examples of objects around the students

		representing the concept of similarity. I explain the definition and conditions of similarity and provide further examples. Subsequently, I illustrate what is meant by corresponding sides and corresponding angles to enhance student understanding.” RTO anticipates common misconceptions that might arise among the students. Consequently, RTO pays close attention to prevalent misunderstandings that could emerge in the students' understanding. She also suggested a solution to prevent the occurrence of these common misconceptions.
	PCK3: The participant explains different approaches to solving a problem, and use of multiple representations.	RTO put forward various approaches to solve the problem, including: employing tracing for drawing, utilizing transformations (such as reflection, translation, and rotation), and applying triangle congruency axioms and theorems. In solving the problem, RTO employed visual-manipulative representations.

All participants recalled prerequisite concepts at the commencement of the lessons outlined in their lesson plans. However, not all the PCK indicators were evident in the produced video lessons. With RTO's video being excluded due to unsuitable video, we observe that HYA and MIF displayed the PCK indicators in alignment with their lesson plans. HYA, however, only demonstrated PCK1 and PCK3 in her video lesson. Notably, she left out the conversation about possible misconceptions, despite having included it in her lesson plan along with a proposed solution to address them. While MIF only exhibits PCK3 in his video. MIF had previously highlighted the relevant prerequisite knowledge for problem-solving, specifically the concept of regular polygons, and had intended to address the measurement of interior angles. However, in the actual video lesson produced by MIF, the discussion focused solely on polygons in a general context. Furthermore, MIF did not engage in any interactive elements during the opening phase. Instead, MIF provided only a verbal explanation of what constitutes a regular polygon to the viewers.

Mathematical knowledge presented by participants and their misconceptions

In terms of mathematical knowledge, all participants accurately solved the problems in their lesson plans and videos. However, HYA and RTO issue misconceptions while solving their problems. HYA managed to demonstrate two indicators of content knowledge, i.e. she solved mathematical problems correctly and used visual mathematical representations effectively. Nonetheless, HYA did not provide deductive and mathematical proof for her statements. Her approach leaned more towards intuition. Both in the lesson plan and video she made, HYA didn’t solve the problem mathematically. She only explains the proof intuitively. Furthermore, she did not effectively communicate her mathematical ideas in her responses. As a result, HYA fulfilled CK1 and CK4 for the content knowledge indicators but did not demonstrate CK2 and CK3. CK1 was displayed through HYA's ability to mention the properties of a square. She employed GeoGebra to plot the given points in the problem, thereby showcasing CK4, as she utilized mathematical representation visually through GeoGebra.

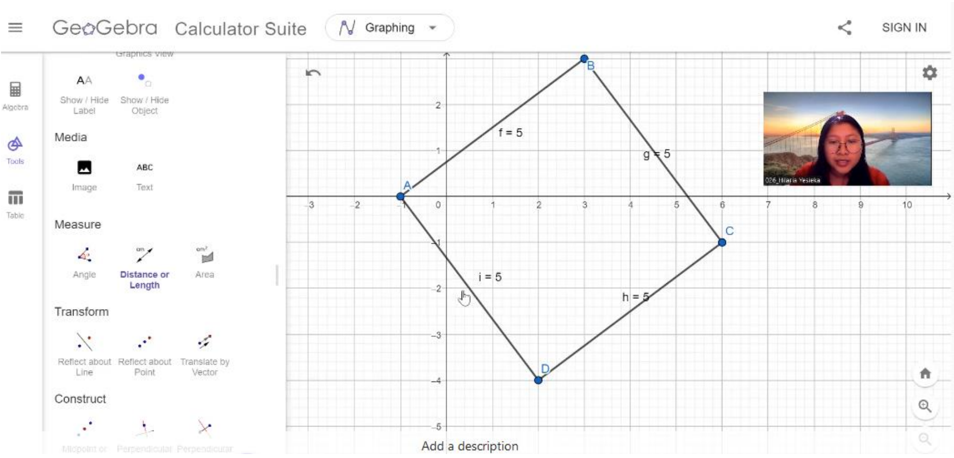
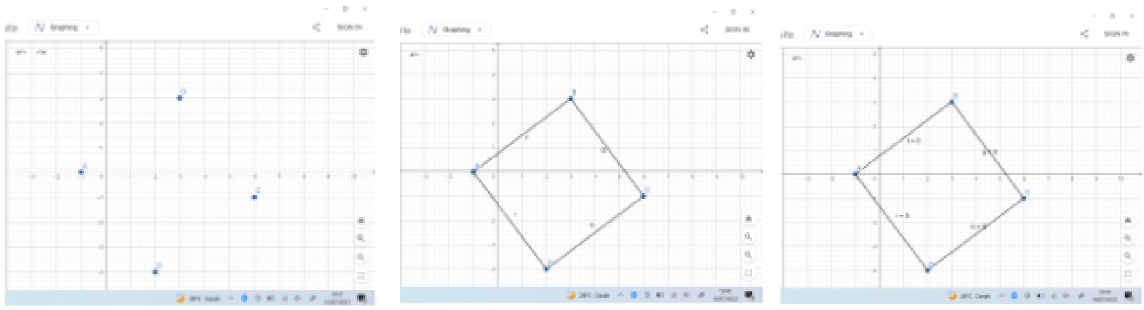


Figure 5. Visual Representation HYA Used through GeoGebra

HYA started by plotting the points that had the coordinates given in the problem into the GeoGebra. Next, she connected the four points to form a figure that resembled a square. In the next step, HYA measured the length of the sides formed by using the distance measurement feature on GeoGebra. The results of this measurement show that the length of each side formed is the same, which is 5 units. From this information, HYA concluded that the quadrilateral shape formed has the properties of a square, one of which is that the sides have the same length.

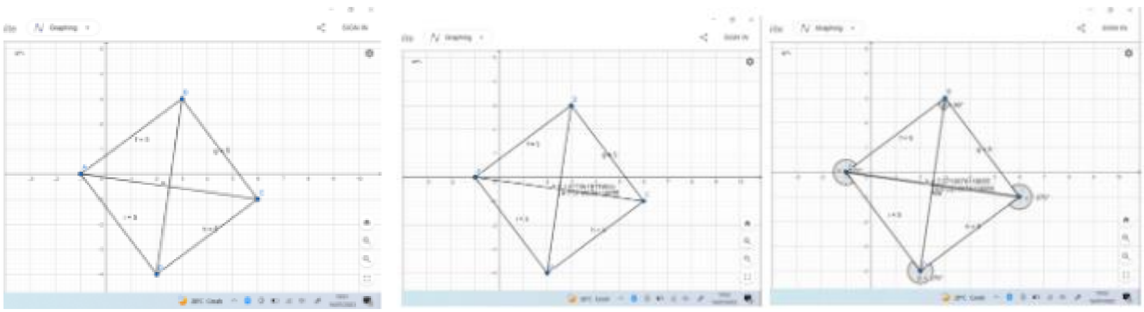


(1) (2) (3)

Figure 6. (1) HYA Plotting the Points, (2) HYA Connects the Points, (3) HYA Measures the Sides

HYA then drew diagonals inside the quadrilateral that had been formed and measured the length of each diagonal using the features available on GeoGebra which she had previously also used to measure the length of the sides. The measurement results show that both diagonals also have the same length. This fact is also a typical property of a square, namely the length of its diagonals is the same.

In the last stage, HYA measured the size of the angle formed by the two sides that meet at the vertex. Using the angle measurement feature provided by GeoGebra, HYA found that each angle formed by the sides of the drawn quadrilateral has the same measure, which is 90°. This is one of the characteristics of a square, where the angles formed by the sides that meet at one point are right angles.



(1) (2) (3)

Figure 7. (1) HYA Draws the Diagonals, (2) HYA Measures the Diagonals' Length, (3) HYA Measures the Angles

Thus, HYA concluded that the quadrilateral formed from the coordinate points given in the problem has equal-length sides, as well as equal-length diagonals, and the angles formed are right angles. Based on this information, HYA concluded that the shape is a square because it fulfills the three typical properties of a square.

The solution outlined by HYA, both in the lesson plan and in the video, showed a lack of effectiveness. This is related to the fact that it is sufficient for HYA to show that if a quadrilateral has equal side lengths and all four angles form right angles, then the quadrilateral is automatically a square. Even without providing evidence that the diagonals are also of the same length, the previous two features are considered sufficient to state that a quadrilateral is a square. Therefore, although the action was acceptable, the lack of understanding of the interconnectedness of the properties of a square caused HYA not to be able to show the CK3 indicator as she should have.

During her problem-solving process, HYA effectively conveyed the sequential actions she took in addressing the given problem to the audience in an intuitive manner. Unfortunately, she did not explain in detail these actions in a formally documented mathematical structure. His explanations remained only in oral form, with no complementary textual representation of how these actions could be translated into mathematical symbols or equations. As a consequence of his reliance on intuitive approaches to problem-solving, HYA had fail in proving the content knowledge indicator CK2, specifically to prove a statement deductively. This limitation manifested in his instructional lesson plan and the instructional video she created.

Although HYA only demonstrated CK1 and CK4 and not CK2 and CK3, her actions were still consistent with the strategies described in her lesson plan and video. However, it should be noted that there were some misconceptions identified in the explanation given by HYA in the video. In her video presentation, HYA illustrated the concept of a rectangle using an eraser; however, these objects are ideally three-dimensional shapes, not two-dimensional shapes. HYA should have clarified that one side of the eraser, which resembles a cube, approximates a rectangular shape. In addition, on another occasion, HYA referred to the y-coordinate as the abscissa, whereas the abscissa is represented by the x-coordinate in the (x, y) coordinate system. y-coordinate is called oordinate. In addition, HYA erred in giving examples of regular polygons. She included rectangles, parallelograms, trapezoids, and rhombi as regular polygons, despite the fact that regular polygons have sides of equal length. In addition, in the introduction to her video, HYA illustrated a rhombus with a picture of a square whose sides are not positioned vertically or horizontally.

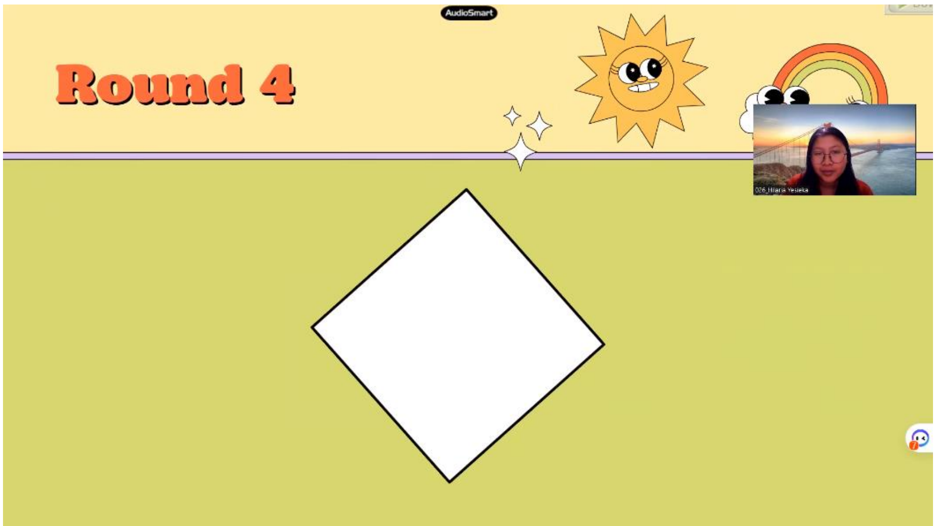
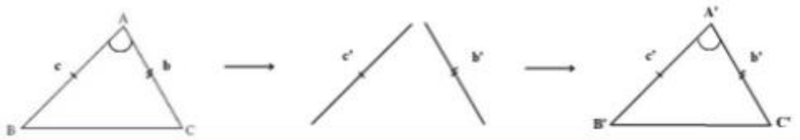


Figure 8. One of HYA's Misconceptions

At this point, it seems that HYA still faces challenges in distinguishing between the geometric concepts of rhombus and square. To address this issue, it would be beneficial for HYA to provide illustrative examples that effectively differentiate between a square and a rhombus, thus eliminating any potential confusion stemming from the overlapping characteristics of the two shapes. Moreover, it should be noted that HYA also indicated on a separate occasion that a square should not be mistaken for a rhombus. However, it is crucial to realize that the properties of a rhombus are inevitably fulfilled by a square, thus highlighting the interconnected nature of these concepts.

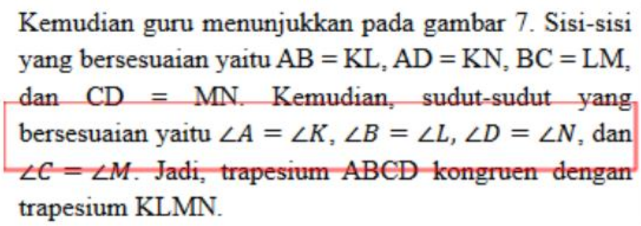
Likewise, RTO managed to solve the given mathematics problem correctly. She showed the indicator of CK1, which is solving math problems correctly. RTO provided various methods to draw a triangle congruent to the given triangle. However, she did not provide a formal proof of the congruence of the triangles she drew. This indicates that RTO did not fulfill the CK2 indicator, namely proving mathematical statements deductively.



Misalkan panjang sisi AB = c dan panjang sisi AC = b maka c' adalah panjang sisi yang bersesuaian dengan c membentuk A'B' dan b' adalah panjang sisi yang bersesuaian dengan b membentuk A'C'. Setelah membuat dua sisi yang bersesuaian sama panjang, selanjutnya menggabungkan kedua sisi tersebut dengan memperhatikan besar $\angle A$ sehingga diperoleh sudut yang bersesuaian yaitu $\angle A'$. Karena terdapat dua sisi dan sudut apit yang bersesuaian dengan ΔABC maka ΔA'B'C' dikatakan kongruen dengan ΔABC.

Figure 9. RTO’s Misconception

Some misconceptions were identified in RTO's lesson plan. In her lesson plan, RTO assumes the length of side AB as c. This is redundant because the symbol AB already indicates the length of the side, so RTO should simply write AB = c. Furthermore, in explaining congruence, the corresponding elements should refer to the sides as line segments. However, here RTO mentions that the corresponding elements are the lengths of the sides. RTO still confuses what is meant by measurement and what is meant by geometry objects, such as sides as line segments. This shows that RTO failed to use mathematical representations well.



Kemudian guru menunjukkan pada gambar 7. Sisi-sisi yang bersesuaian yaitu AB = KL, AD = KN, BC = LM, dan CD = MN. Kemudian, sudut-sudut yang bersesuaian yaitu $\angle A = \angle K$, $\angle B = \angle L$, $\angle D = \angle N$, dan $\angle C = \angle M$. Jadi, trapesium ABCD kongruen dengan trapesium KLMN.

Figure 10. Another RTO's Misconception

Consistent with the previous misconception, RTO also mixed up symbols representing angles and symbols representing angle measurements. Here, RTO used the equals sign to show congruence, while the equals symbol should only be used to show equality in measurement.

Unlike the others, MIF showed all indicators of CK comprehensively. He managed to solve the problem correctly. The answer she gave also showed that MIF addressed the problem deductively. However, MIF did not present the mathematical solution sequentially. Instead, she explained the steps narratively, aligning them with the instructional steps outlined in the lesson plan. From the steps outlined, there were no errors in the use of mathematical representations. The video produced by MIF was in accordance with what he had written in the lesson plan.

Integration of Mathematical Action Technology in Learning Videos Made by Prospective Teachers

This section presents how prospective mathematics education teachers used the mathematics action technology (MAT) GeoGebra in their online learning videos. Of the three subjects, only MIF and HYA whose learning videos could be analyzed further. Meanwhile, RTO only sent a recording of how to explain the answer without explaining the answer to the students. Recall that all levels of implementation of the math action tool occurred in the online learning video (level 0), as shown in Figure 11.

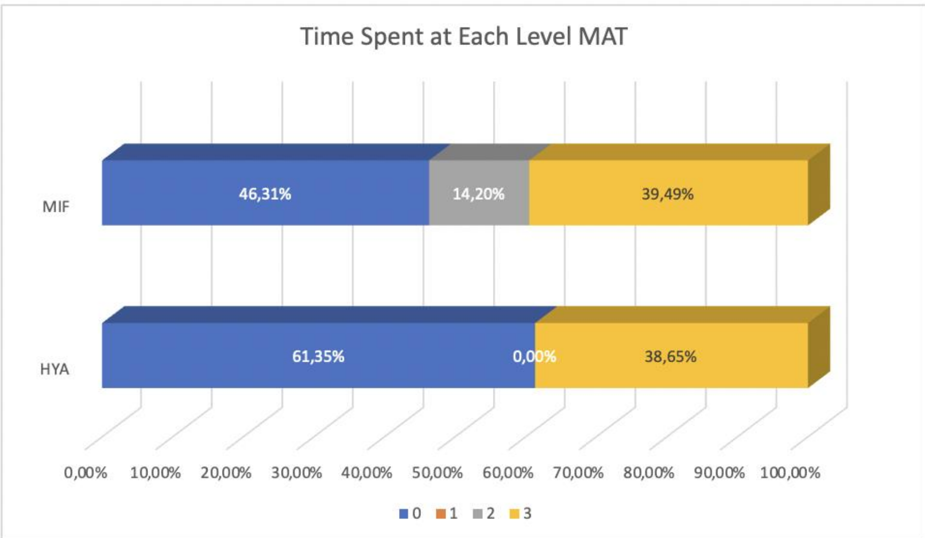


Figure 11.

All participants in this study started their learning from level 0. To clarify and simplify the instructions, they chose PowerPoint as the medium to introduce the task to the virtual students, as shown in Figure xx. Interestingly, each participant varied in the use of MAT. HYA showed consistency in her method by using zero-level for more than 60% of the duration of their video lessons. Meanwhile, MIF opted for a more diverse approach using level 0 MAT for 46% of the total duration. However, when using GeoGebra as a level 3 MAT, MIF and HYA showed similarities in their application, with 39.49% and 38.65% of the duration, respectively. What was surprising was that HYA, despite using level 0 tools frequently, utilized GeoGebra sparingly for MAT level 2. Meanwhile, MIF utilized GeoGebra at level 2 for 14.20% of their online learning duration. These results show that each participant has different approaches and tool preferences to facilitate online learning.

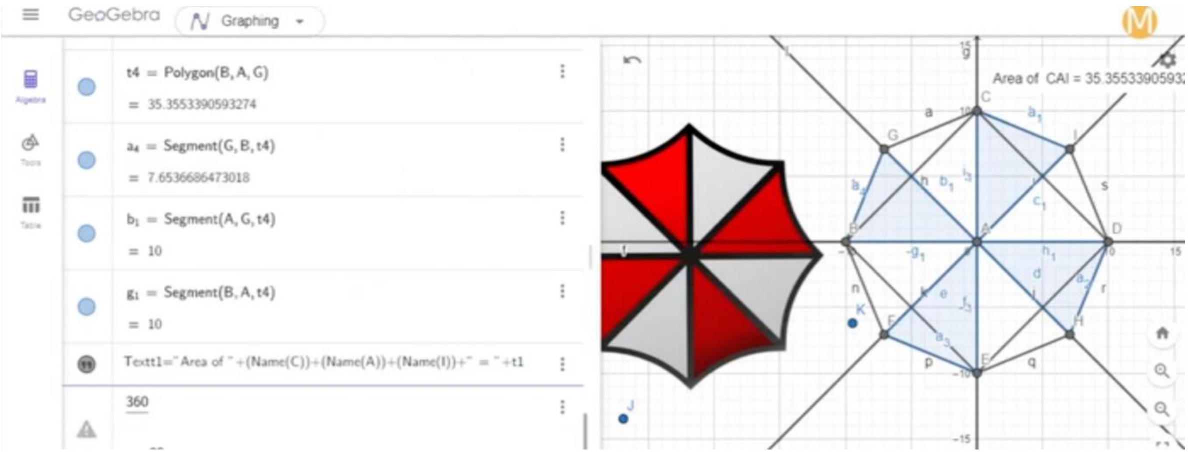


Figure 12. MIF Explanation on Mathematics Problem Using GeoGebra

MIF and HYA used the math action tool at level 3 for the same purpose. Moreover, MIF used level 3 to prepare the proof by intuitively showing the magnitude of the angle on the polygon (See Figure xx). HYA also used GeoGebra to show the construction of the building. Using dynamic features in such constructions can increase students' awareness of the essential elements to start the proof. However, the argumentation of the proof required to solve this problem is done in the last five minutes of the video using screenshots of the GeoGebra construction (See Figure xx) (i.e. level 1).

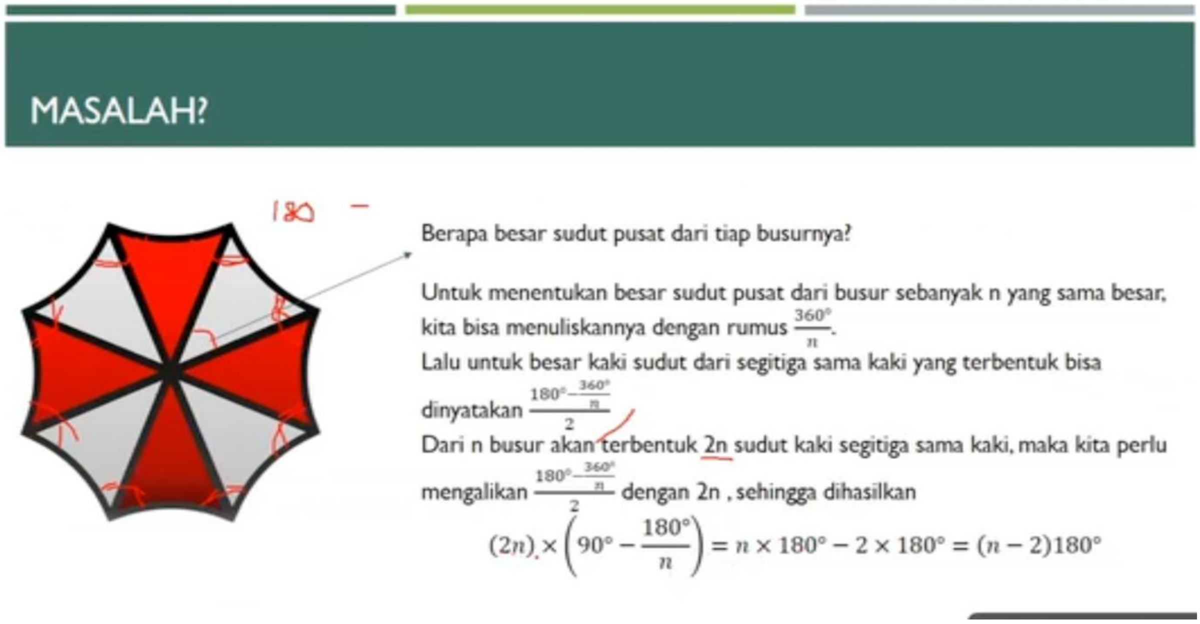


Figure 13. Proof explanation by MIF using GeoGebra Screenshot

The results of this study on each prospective teacher's approach and tool preferences in the online learning process provide important insights into the diversity of strategies in mathematics teaching. Mathematical Action Technology (MAT) as a tool designed to support mathematical activities can be an important instrument to enrich the learning process (Bonafini & Lee, 2021; Paidican & Arredondo, 2022; Pea et al., 1987). However, its application in the online learning environment significantly varies among educators. Some educators, such as HYA as a prospective teacher, tend to focus on level 0 tools, which may be considered more basic but essential, suggesting a preference for building a strong foundation before moving on to more complex tools (Luik et al., 2018). Conversely, MIF, by combining the use of level 0 tools and GeoGebra in MAT levels 2 and 3, indicates a more holistic and dynamic approach to integrating technology in teaching.

GeoGebra, as MAT levels 2 and 3, allows for interactivity and deeper exploration of mathematical concepts. Its use by some educators indicates a desire to enrich the learning process with tools that can stimulate critical thinking and concept understanding through visualization and simulation (Kelly, 2010). Overall, the variation in educators' use of tools and MAT suggests that no one "best" approach applies to all situations (Conrad, 2019; Guerrero & Crites, 2013). Each educator has their teaching philosophy and style, which influences the tools and technologies they use (Kadıoğlu-Akbulut et al., 2023). It underscores the importance of freedom for educators to choose and adapt tools that best suit their needs and teaching style in an online learning context.

Conclusion

In the participants' lesson plans and the learning videos they created, there was effective integration of technology. They skilfully used technology as a means to deliver educational content and used GeoGebra as a mathematics action technology to augment the learning experience. In addition, the participants skilfully showcased their pedagogical methods and mathematical representations, effectively utilizing technology to present both static and dynamic visual representations. This underscores the importance for educators to comprehensively develop their knowledge across the components of the TPACK framework, as emphasized by experts in the field. While it is true that all participants successfully solved the mathematical problems in their lesson plans and learning videos, there were some instances where adherence to mathematical rigor and precision was lacking. In particular, there were some misconceptions that emerged among some participants, potentially leading students to make mathematical errors.

The mathematical tools used by the participants in their lesson plans and learning videos had a dual role, as amplifiers and reorganizers at various times. In cases where the mathematical tools served as amplifiers, they demonstrated the ability to enrich lessons with more precise mathematical representations that would otherwise be challenging and time-consuming to create manually by hand. In this context, mathematical tools serve to enhance, accelerate, and streamline an aspiring educator's existing proficiency in solving mathematical problems. Conversely, when mathematical aids are used as reorganizers, they have the potential to reshape the way mathematics is presented to students. This transformative aspect provides prospective teachers with the opportunity to reconfigure their understanding of mathematical knowledge for pedagogical purposes (Bonafini & Lee, 2021).

Similarly, when a prospective teacher chooses to use mathematical tools as reorganizers, their lesson gains the potential to revolutionize the way in which mathematical concepts are introduced and organized for students. This transformation paves the way for students to potentially gain a new and deeper understanding of mathematics (Bonafini & Lee, 2021). Essentially, the utilization of mathematical technology serves as a catalyst to reshape the knowledge construction process.

The process of teaching mathematics through technology has the potential to contribute to the TPACK balance of prospective teachers. The experiences and insights gained by prospective teachers during their time as students can significantly influence their later choices regarding the integration of technology into their future teaching endeavours. Giving pre-service teachers the opportunity to create lesson plans and learning videos that integrate technology, reflects their future professional practice, where they will have to make informed decisions regarding the most appropriate technology to enhance the teaching of specific mathematical concepts.

The data analysis presented in this study used the TPACK framework and the resetting/reinforcing lens to understand how prospective mathematics education teachers used their mathematical, pedagogical, and technological knowledge in their video lessons by integrating GeoGebra. This research can be further extended by using pre-service mathematics education teacher students who have already practiced in schools. It is because prospective mathematics education teachers who have never practiced in schools only use mathematics action tools to introduce the material, not to construct or invite students to explore together and provide immediate feedback. For future research, we can analyze the TPACK of prospective mathematics teachers, both those who have not and who have practiced in schools using GeoGebra Classroom. The platform allows teachers to conduct formative assessments to determine student mastery of the material in real-time.

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